Thornthwaite equation

Description

The Thornthwaite equation is a method developed by Thornthwaite (1948) based on an empirical approach in order to estimate potential evapotransipiration. Evapotranspiration is originally destinated to be computed on a monthly basis but daily estimations are possible as well (cf. Thornthwaite & Mather (1957).

Formula

The potential evapotranspiration according to the Thornthwaite (1948) formulation PET_{Thorn} [mm/day] is calculated as follows (the equation for T > 26 is an approximation of the original table proposed by Willmott et al. (1985) :

$$PET_{Thorn} = egin{cases} 0, & ext{if } T < 0 \ 16 \cdot rac{N}{360} \cdot \left(rac{10 \cdot T}{I}
ight)^a, & ext{if } 0 \leqslant T \leqslant 26 \ rac{N}{360} \cdot \left(-415.85 + 30.5332.24 \cdot T - 0.43 \cdot T^2
ight), & ext{if } T > 26 \end{cases}$$

where N is the duration of sunlight in hours, varying with season and latitude (cf. calculation of daylight hours after the FAO formulation, or Table 8 in Thornthwaite & Mather (1957), T average daily air temperature [°C], and I heat index calculated as follows:

$$I = \sum_{Jan}^{Dec} \left(\frac{max[0,T_m]}{5}\right)^{1.514}$$

where T_m is monthly mean temperature [°C].

The exponent a is calculated as follows:

$$a = (6.75 \cdot 10^{-7} \cdot I^3) - (7.71 \cdot 10^{-5} \cdot I^2) + (0.01792 \cdot I) + (0.49239)$$

An improved version of the daily Thornthwaite equation was proposed by Camargo et al. (1999) and Pereira & Pruitt (2004). Instead of average daily air temperature T [°C], Camargo et al. (1999) suggested to use an "effective temperature" T_{ef} [°C] given by:

$$T_{ef} = rac{1}{2} \cdot k \cdot (3 \cdot T_{max} - T_{min})$$

where k = 0.72, T_{max} is maximum daily temperature [°C], and T_{min} minimum daily temperature [°C].

Pereira & Pruitt (2004) corrected this equation by including a day-night ratio:

$$T_{ef}^* = T_{ef} \cdot \frac{N}{24 - N}$$

$$\text{with}\, \frac{T_{max}+T_{min}}{2} \leqslant T_{ef}^* \leqslant T_{max}$$

where $T_{e\!f}^{*}$ [°C] is the corrected $T_{e\!f}$, and N the daylight hours.

Reference

Thornthwaite (1948) Thornthwaite & Mather (1957) Willmott et al. (1985) Camargo et al. (1999) Pereira & Pruitt (2004)

Penman equation

Description

Penman (1948) developed an equation based on a more theoretical approach than Thornthwaite (1948) in order to estimate potential evaporation from open water. His equation has also been widely used to estimate evapotranspirations from vegetation covers. Shuttleworth (1993) reformulated the Penman equation in metric units.

Formula

The potential evapotranspiration after Penman (1948) PET_{Pen} [mm/day] is calculated as follows (Shuttleworth 1993):

$$PET_{Pen} = rac{\Delta}{\Delta + \gamma} \cdot rac{R_n}{\lambda} + rac{\gamma}{\Delta + \gamma} \cdot rac{6.43 \cdot (1 + 0.536 \cdot U) \cdot \Delta e}{\lambda}$$

where Δ is the the slope of the saturation vapor pressure curve [kPa/ŰC], γ the psychrometric constant [kPa/°C], R_n the net radiation [MJ·m⁻²·d⁻¹], λ the latent heat of vaporization, U the wind speed [m/s], and Δe the vapor pressure deficit [kPa].

The slope of the saturation vapor pressure Δ is calculated as follows (Allen et al. 1998):

$$\Delta = \frac{4098 \cdot 0.6108 \cdot e^{\frac{17.27 \cdot T}{T+237.3}}}{(T+237.3)^2}$$

where T is air temperature [°C].

The latent heat of vaporization λ [MJ/kg] is calculated as follows (Shuttleworth 1993):

$$\lambda = 2.501 - 0.002361 \cdot T$$

The psychrometric constant γ [kPa/°C] is calculated as follows (Shuttleworth 1993):

$$\gamma = 0.0016286 \cdot \frac{p_{atm}}{\lambda}$$

where p_{atm} is the atmospheric pressure [kPa] and calculated as follows (Allen et al. 1998):

$$p_{atm} = 101.3 \cdot \left(\frac{293 - 0.0065 \cdot z}{293}\right)^{5.26}$$

where z is elevation above sea level [m].

References

Penman (1948) Shuttleworth (1993) Allen et al. (1998)

The original document is available at http://wiki.fire.wsl.ch//tiki-index.php?page=Potential+evapotranspiration